Drewno 2017, Vol. 60, No. 199
DOI: 10.12841/wood.1644-3985.192.02

Piotr Rapp

THE NUMERICAL MODELING OF ADHESIVE JOINTS IN REINFORCEMENT OF WOODEN ELEMENTS, SUBJECTED TO BENDING AND SHEARING

The subject of this paper is a formulation and discussion about an adhesive joint model in cases of reinforcement or reconstruction of weakened cross-sections of wooden elements. The problem is modeled within the linear theory of elasticity as a plane stress case. It is assumed that wood is an orthotropic material. Reinforcement is achieved by attaching a covering plate, and reconstruction by introduction of an insert at a weakened (deteriorated) zone of the element. Analysis of the influence of covering plates and insert thickness on the stress state in adhesive and in adherends is carried out. Elements subjected to bending moments or bending moments with shear forces are considered.

Keywords: wood, adhesive joints, element reinforcement, reconstruction of weakened cross-section, stress concentration, numerical modeling

Introduction

An adhesive joint is an assembly of two plane stress elements connected at a common surface by an adhesive. It is assumed that the adherends and the adhesive are of constant or moderately varying thickness.

An adhesive joint is modeled as a plane two-dimensional assembly parallel to the plane 0XY in the Cartesian set of co-ordinates 0XYZ. Both adherends and an adhesive projected onto the plane 0XY form the same figure of an arbitrary shape.

It is assumed that effects of bending in adherends are negligible – they are not taken into account. Thus, it was further assumed, that stresses are constant across an adherend thickness and form plane stress states parallel to the plane 0XY. The layout of an adhesive joint is presented in figure 1.

The adherends 1 and 2 have thickness described by functions $g_1 = g_1(x, y)$ and $g_2 = g_2(x, y)$. The mid-plane of the adhesive is given by the function $s = s(x, y)$. The thickness of adhesive $t = t(x, y)$ is always greater than zero.

Piotr Rapp✉ (*piotrrapp@gmail.com*), Wood Technology Institute, Poznań, Poland

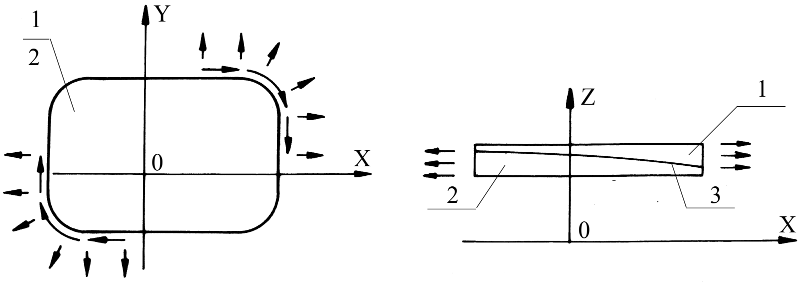


Fig. 1. Layout of adhesive joint: 1 – adherend 1, 2 – adherend 2, 3 – adhesive

Adherends are made of orthotropic materials, with the principal axes of orthotropy coinciding with the axes X and Y. An orthotropic material in a plane stress state is described by the moduli of longitudinal elasticity E_{kx} , E_{ky} , the shear moduli G_{kxy} and Poisson's ratios ν_{kxy} , ν_{kyx} (where k = 1, 2).

Adhesive is modelled as an isotropic linear-elastic medium characterised by the material parameters: Young's modulus E_s, shear modulus G_s and Poisson's ratio ν_s , where $E_s = 2(1 + \nu_s)G_s$. Adhesive is subjected to the stresses $\tau_x = \tau_x(x,y)$, $\tau_y = \tau_y(x, y)$ tangent to the mid-plane and the stress $\sigma_N = \sigma_N (x, y)$ normal to this plane. These stresses are assumed to be constant across the adhesive thickness.

If an adherend thickness is larger than zero on its edge, then the edge is called *non-sharp*. Stresses acting at non-sharp edges of an adherend k are denoted by p_{kx} and p_{ky} (k = 1, 2). It is assumed that the stresses p_{kx} and p_{ky} are parallel to the axes X and Y, respectively, and that they are constant across the adherends thickness. They are treated as a given external loading acting on adherends in the plane parallel to 0XY. The adherend thickness along the entire edge or on its fragment can be zero. In that case the edge is called *sharp* [Rapp 2015a, 2016].

If the external surfaces of an adherend forming the edge K intersect at an angle $\alpha > 0$, then the edge K of the adherend is called *obtuse sharp edge* (fig. 2a). If the external surfaces of the adherend forming the sharp edge K are mutually tangent ($\alpha = 0$), then the edge K is called *tangential sharp edge* (fig. 2b). No edge loading is defined along a sharp edge.

a) b)

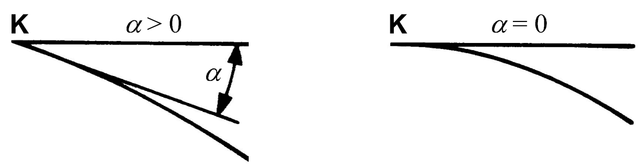


Fig. 2. Cross-sections of two types of sharp edges: a) obtuse sharp edge K, b) tangential sharp edge K

Displacements in the adherends 1 and 2 are defined by the functions $u_1 = u_1(x, y)$ and $u_2 = u_2(x, y)$ for the direction X and the functions $\upsilon_1 = \upsilon_1(x, y)$ and $\upsilon_2 = \upsilon_2(x, y)$ for the direction Y. The displacements u_1, u_2, υ_1, υ_2 are considered as unknowns. Equations of the theory of elasticity expressed in displacements with boundary conditions for the plane stress state have been formulated in the paper [Rapp 2015a]. Knowing the displacements functions u_1, u_2, υ_1, υ_2 one can determine the stress and strain states in the adhesive and the adherends.

An overview of problems related to the reinforcement or reconstruction of deteriorated elements by means of adhesive joints in various branches of technology is presented in the following literature [Ahn and Springer 2000; Bahei-El-Din and Dvorak 2001; Boss et al. 2003; Kaye and Heller 2002; Kumar et al. 2006; Rapp 2015b].

The present paper can be viewed as a continuation of the analysis presented in [Rapp 2016], where the problem of axially loaded adherends was addressed.

Materials and methods

It is assumed, that adherend 2 is subjected to a bending moment **M** (fig. 3a) or a moment **M** together with a shear force **T** (fig. 3b) acting in the plane 0XY. Adherend 1 is not loaded.

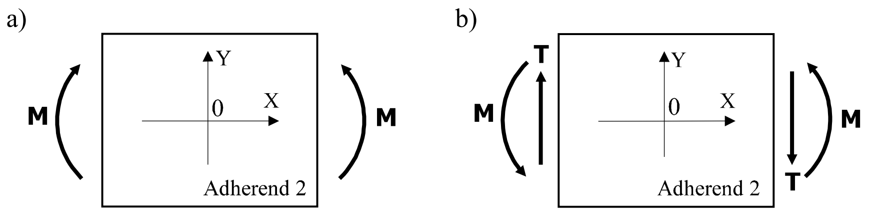


Fig. 3. Loading of the adherend 2: a) moment M, b) moment M with shear force T

If adherend 1 has been attached to adherend 2 of a constant thickness, then the total thickness of both adherends is greater than that of adherend 2. Such an adhesive joint is a reinforcement of adherend 2 with a *covering plate* (adherend 1). Such types of reinforcement of adherend 2 with covering plates with various geometry of edges are presented in figure 4.

If there is a local loss of material in adherend 2, than new material can be inserted to restore the original thickness of adherend 2. Such an adhesive joint is considered as a reconstruction of adherend 2 with an *insert* (adherend 1). Variants of such a reconstruction with various types of inserts are presented in figure 5.

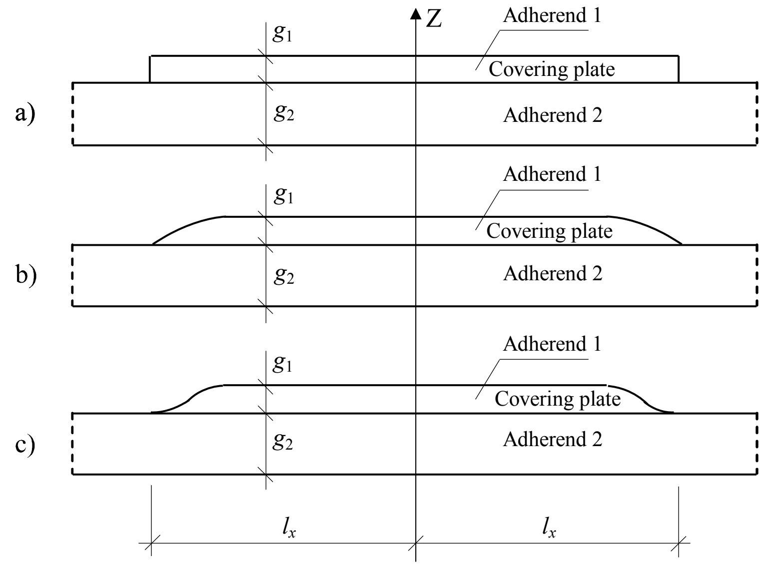


Fig. 4. Variants of reinforcement of adherend 2 by covering plates: a) constant thickness covering plate, b) covering plate with obtuse sharp edges, c) covering plate with tangential sharp edges

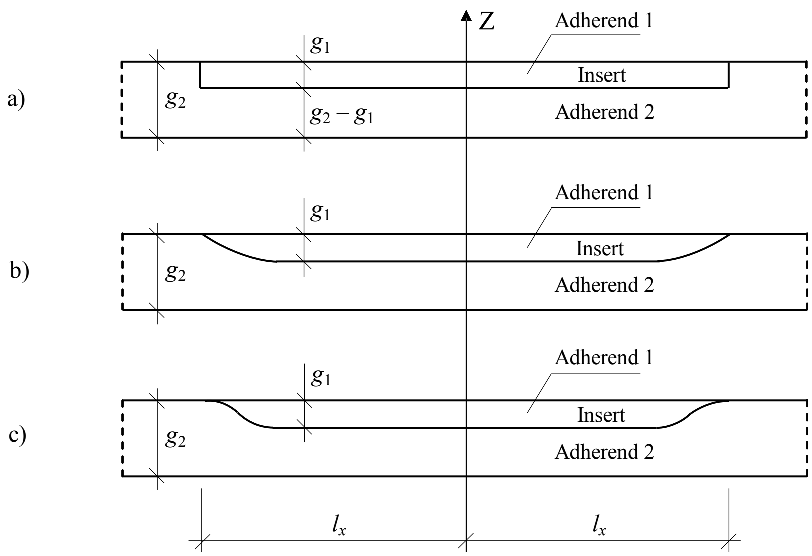


Fig. 5. Variants of reconstruction of adherend 2 with inserts: a) constant thickness insert, b) insert with obtuse sharp edges, c) insert with tangential sharp edges

Anchoring zones of covering plates and inserts should be short. Stresses in both adherends between the anchoring zones should be uniform and constant and stresses in the adhesive equal to zero. There should be no stress concentrations in the adhesive.

Meeting the above conditions depends strongly on edge types and varying thickness of covering plates and inserts in anchoring zones.

This paper is devoted to the analysis of this influence on stress states in the adhesive and the adherends and to a proper choice of an anchoring zone range.

Adhesive joints analysed in the paper may be used to restore wooden structures in historic buildings. Figure 6 shows an example of attic beams reconstruction, where dark spots represent the original substance of beams and the light elements are the layers of new wood used to restore the original cross--section of the beams [Rapp 2015b].

Fig. 6. An example of attic beams reconstruction

The equations for the displacements u_1, u_2, υ_1, υ_2 form a set of four elliptic partial differential equations of the second order. The existence and uniqueness of the solution to the set of elliptic equations with appropriate boundary conditions are ensured [Fichera 1972].

The boundary value problems in displacements are solved here, using the classical finite-difference method [Forsythe and Wasow 1960; Cea 1964]. The method is based on a replacement of differential operators with difference operators defined in a discrete set of points (nodes), which are intersections of lines forming a difference mesh in a rectangle $2l_x \times 2l_y$.

The difference mesh has a regular rectangular shape with side lengths Δx and Δy. There are m nodes in the direction X ($j = 1, 2, ..., m$), and n nodes in the direction Y ($i = 1, 2, ..., n$), with $n, m \geq 5$. It is assumed that n and m are odd numbers. The unknowns in the finite-difference method are the values of the displacement functions $u_{kr,s} = u_k(x_r, y_s)$ and $\upsilon_{kr,s} = \upsilon_k(x_r, y_s)$ for $k = 1, 2$ or the values of the shear stress functions $\tau_{xr,s} = \tau_x(x_r, y_s)$ and $\tau_{yr,s} = \tau_y(x_r, y_s)$ in the adhesive defined in the nodes of the finite-difference mesh. Derivatives of functions are approximated with central differences.

Displacement equations are formulated for all the nodes of the finite--difference mesh, excluding those, where kinematic boundary conditions are defined and those at sharp edges. In the case of nodes with prescribed kinematic boundary conditions, if they are constrained, zero displacements are substituted. For nodes on sharp edges, static boundary conditions are applied. The application of central differences to nodes at the edges, with the exception of sharp ones, results in fictitious values of unknown functions for nodes falling out of the rectangular domain $2l_x \times 2l_y$. Those fictitious values of the unknown functions are eliminated by means of static boundary conditions for non-sharp edges. In the case of sharp edges, the fictitious nodes beyond the rectangular domain $2l_x \times 2l_y$ are not introduced. For internal nodes at sharp edges, central differences are used for the direction along the edge, while for the direction across edges and for corner nodes unilateral differences spanning three nodes in the direction X and Y are used. A complete set of linear equations by the finite difference method in terms of displacements consists of $4nm$ equations. The matrix formed from coefficients of equations is not symmetric and is singular because the adhesive joint itself is a mechanism. Non-singularity of the matrix and uniqueness of the solution for a system expressed in terms of displacements is obtained, if kinematic boundary conditions for displacements u_k and v_k are imposed to make the adhesive joint geometrically stable.

Test computations indicate, that the finite-difference meshes from the range $41 \leq m, n \leq 51$ yield a relative error of solution not exceeding 0.5%.

Results and discussion

It is assumed that the entire loading is applied at the edges $x = \pm l_x$ of the adherend 2. Adherends 1 and 2 carry the loading together in the range $-l_x < x < l_x$. Stresses in the adhesive are relatively large in the areas near the edges $x = \pm l_x$. These areas of the joint are considered as anchoring zones of a covering plate or an insert (fig. 19). In these areas, the adhesive joint carries an appropriately large, assumed portion of the loading.

Stress distributions in anchoring zones depend on the adherends thickness at the edges $x = \pm l_x$. It was assumed in the analysis, that covering plates and inserts can have a constant thickness g_1 = const, as in figures 4a and 5a or varying thickness $g_1(x, y)$ with obtuse sharp edges – figures 4b and 5b or with tangential sharp edges – figures 4c and 5c. Curvilinear shapes of covering plates and inserts are assumed in a parabolic form [Rapp 2016].

An adhesive joint consisting of two flat wooden adherends with dimensions $2l_x \times 2l_y$ = 10.0 cm × 8.0 cm and g_1 = 0.2 cm, g_2 = 1 cm is considered.

It is assumed that the principal axes of wood orthotropy coincide with the direction X parallel to the wood grain and the radial direction Y perpendicular to the wood grain. Material properties were assumed for spruce [Neuhaus 1994]:

– modulus of elasticity parallel to grain $E_x = 1.2 \cdot 10^6$ N/cm^2,

– modulus of elasticity perpendicular to grain $E_y = 0.8 \cdot 10^5$ N/cm^2,

– shear modulus $G_{xy} = 0.6 \cdot 10^5$ N/cm^2,

– Poisson's ratios $v_{xy} = 0.03$, $v_{yx} = 0.45$ (notation of v_{xy}, v_{yx} by Rapp [2015]).

Material parameters for adhesive were assumed as: $G_s = 45000$ N/cm^2, $E_s = 121500$ N/cm^2, $v_s = 0.35$ and the thickness $t = 0.04$ cm.

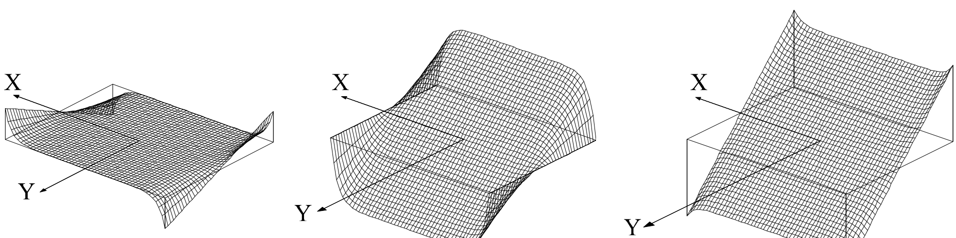
The loading of adherend 2 causes stresses τ_x, τ_y, σ_N in the adhesive and plane stress states in the adherends with σ_{kx}, σ_{ky}, τ_{kxy} ($k = 1, 2$).

In particular cases, some of these stresses are dominant in that they play the main role in carrying loading.

For the joints presented in figures 4 and 5 the loading form shown in figure 3 was assumed and the resulting two-dimensional boundary value problem was solved using the finite difference method using the extended precision format.

For a constant moment M (fig. 3a) the stress n_x in adhesive (n_x it is the resultant of the shear stress τ_x and normal stress σ_N) and normal stresses σ_{1x}, σ_{2x} in adherends are dominant.

Distributions and values of the stress n_x in adhesive and the stresses σ_{1x}, σ_{2x} in adherends for the loading M = 10 N·cm are shown in figures 7-11. It was assumed that the moments are applied to the edges of adherend 2 as linearly distributed normal stress. No graphs for the example from figure 5a are presented because they only differ from those in figure 7a by values, featuring the same shape. The part of the adhesive joint over the X axis (fig. 3a) is subjected to compression, and under the X axis – to tension in the form of

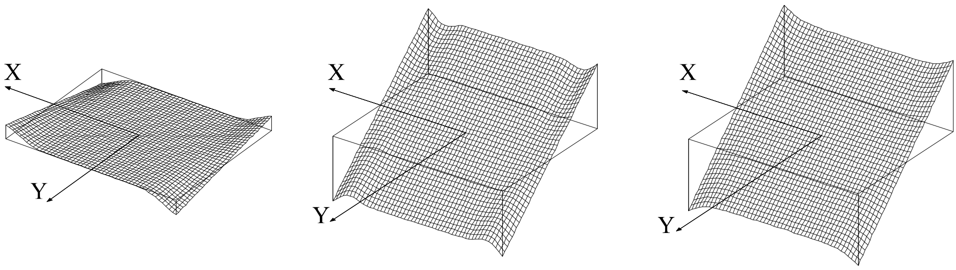


a) Stress n_x in adhesive
max $|n_x|$ = 0.35724 N/cm^2

b) Stress σ_{1x} in adherend 1
$\sigma_{1x}(0, \pm l_y) = \pm 0.78146$ N/cm^2

c) Stress σ_{2x} in adherend 2
$|\sigma_{2x}(\pm l_x, \pm l_y)| = 0.9375$ N/cm^2
$\sigma_{2x}(0, \pm l_y) = \pm 0.78148$ N/cm^2

Fig. 7. Stresses in adhesive joint with covering plate of constant thickness (fig. 4a) subjected to M = 10 N·cm

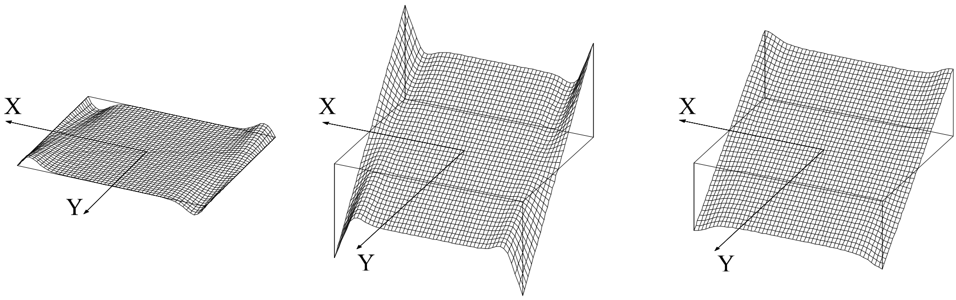


a) Stress n_x in adhesive
max $|n_x| = 0.16992$ N/cm^2

b) Stress σ_{1x} in adherend 1
$|\sigma_{1x}(\pm l_x, \pm l_y)| = 0.85109$ N/cm^2
$\sigma_{1x}(0, \pm l_y) = \pm 0.78219$ N/cm^2
min $|\sigma_{1x}| = 0.70829$ N/cm^2

c) Stress σ_{2x} in adherend 2
$|\sigma_{2x}(\pm l_x, \pm l_y)| = 0.9375$ N/cm^2
$\sigma_{2x}(0, \pm l_y) = \pm 0.78223$ N/cm^2

Fig. 8. Stresses in adhesive joint with covering plate of varying thickness and obtuse sharp edges (fig. 4b) subjected to M = 10 N·cm

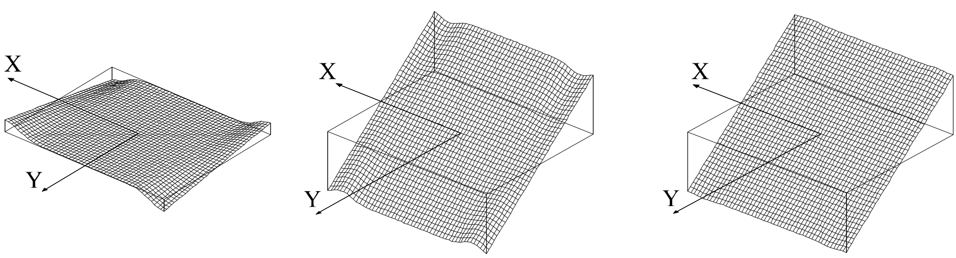


a) Stress n_x in adhesive
max $|n_x| = 0.14513$ N/cm^2

b) Stress σ_{1x} in adherend 1
$|\sigma_{1x}(\pm l_x, \pm l_y)| = 1.3230$ N/cm^2
$\sigma_{1x}(0, \pm l_y) = \pm 0.78388$ N/cm^2
min $|\sigma_{1x}| = 0.70055$ N/cm^2

c) Stress σ_{2x} in adherend 2
$|\sigma_{2x}(\pm l_x, \pm l_y)| = 0.9375$ N/cm^2
$\sigma_{2x}(0, \pm l_y) = \pm 0.7839$ N/cm^2

Fig. 9. Stresses in adhesive joint with covering plate of varying thickness and tangential sharp edges (fig. 4c) subjected to M = 10 N·cm

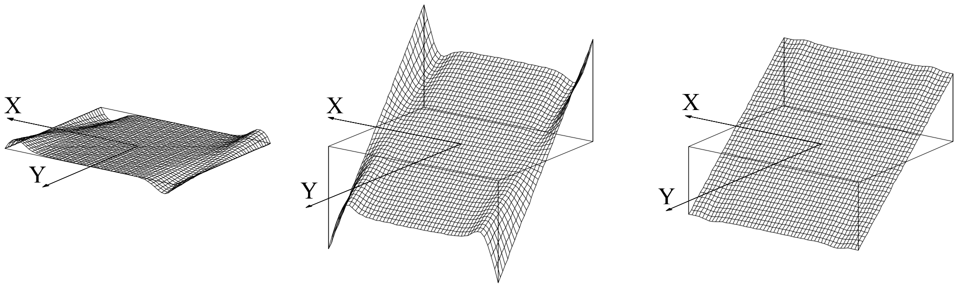


a) Stress n_x in adhesive
max $|n_x| = 0.17720$ N/cm^2

b) Stress σ_{1x} in adherend 1
$|\sigma_{1x}(\pm l_x, \pm l_y)| = 0.90436$ N/cm^2
$\sigma_{1x}(0, \pm l_y) = \pm 0.93751$ N/cm^2
min $|\sigma_{1x}| = 0.82899$ N/cm^2

c) Stress σ_{2x} in adherend 2
$|\sigma_{2x}(\pm l_x, \pm l_y)| = 0.9375$ N/cm^2
$\sigma_{2x}(0, \pm l_y) = \pm 0.93755$ N/cm^2
max $|\sigma_{2x}| = 0.96065$ N/cm^2

Fig. 10. Stresses in adhesive joint with insert of varying thickness and obtuse sharp edges (fig. 5b) subjected to M = 10 N·cm



a) Stress n_x in adhesive
max $|n_x| = 0.16028$ N/cm^2

b) Stress σ_{1x} in adherend 1
$|\sigma_{1x}(\pm l_x, \pm l_y)| = 1.4838$ N/cm^2
$\sigma_{1x}(0, \pm l_y) = \pm 0.93778$ N/cm^2
min $|\sigma_{1x}| = 0.80164$ N/cm^2

c) Stress σ_{2x} in adherend 2
$|\sigma_{2x}(\pm l_x, \pm l_y)| = 0.9375$ N/cm^2
$\sigma_{2x}(0, \pm l_y) = \pm 0.93784$ N/cm^2
max $|\sigma_{2x}| = 0.96852$ N/cm^2

Fig. 11. Stresses in adhesive joint with insert of varying thickness and tangential sharp edges (fig. 5c) subjected to M = 10 N·cm

linearly distributed normal stress. In adhesive joints loaded by moments, in the tensile and compressive fragments, stress functions along the X axis in adhesive and adherends for each set value $y \neq 0$ are similar to the corresponding stress functions in adhesive joints loaded axially. Thus, for the adhesive joints loaded by moments, the width of anchoring zone can be assessed using the same expressions as in the joints loaded axially, as given in [Rapp 2016].

Normal stresses between anchoring zones are approximately equal in both adherends, they are linearly distributed along the Y axis and constant along the X axis.

In order to assess the effectiveness of reinforcement or reconstruction of the adherend 2 using the adherend 1, the normal stresses σ_{1x} and σ_{2x} in adherends 1 and 2 were computed and compared with the stresses, which would occur in uniform elements equivalent to the connected adherends 1 and 2. It was concluded, that the stresses for the elements with covering plates and inserts differ insignificantly (not more than ±0,339% in the analysed cases) from the ones in the uniform elements. A slight difference, with an error within a range of ±0.01 to ±0.05%, is obtained for the case with an insert with varying thickness and obtuse sharp edges.

In the section $-l_x + l_{\mathrm{anch}} \leq x \leq l_x - l_{\mathrm{anch}}$, i.e. in the region of reinforcement or reconstruction, the analysed adhesive joints carry the bending moment in a way very similar to that for a uniform element with dimensions equal to the sum of the dimensions of adherends 1 and 2. The closer this similarity in the reinforcement or reconstruction zone, the smaller the flexibility of the adhesive.

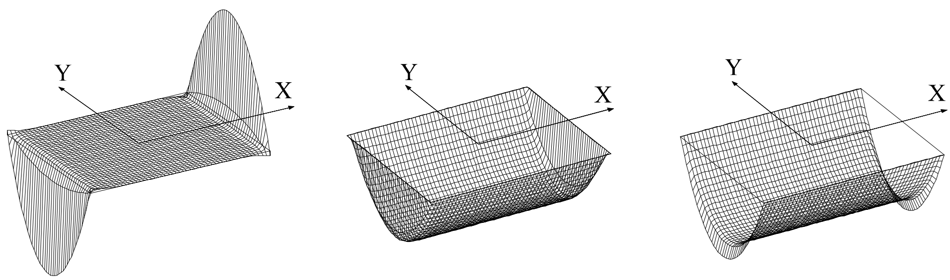
The second analysed problem is related to the way in which adhesive joints with covering plates or inserts, loaded according to figure 3b are subjected to a shear force T. In order to yield a constant shear force in cross-sections parallel to the Y axis , loading of an adhesive joint has to form a couple ±T. The resulting moment equal to $-2\mathsf{T}l_x$ has to be in equilibrium with another moment of the value $2\mathsf{T}l_x$. This condition can be met in many ways. The best possibility is to adopt the stresses τ_{1xy} and τ_{2xy} in adherends resulting mainly from the shear force T, i.e. not depending or depending insignificantly on the remaining loading acting on the adhesive joint. That is why the moment $-2\mathsf{T}l_x$ resulting from the couple $\pm\mathsf{T}$ was equilibrated by two moments $\mathsf{M} = \mathsf{T}l_x$ Then the normal stresses σ_{1x} and σ_{2x} result, which are not superimposed onto the shear stresses τ_{1xy} and τ_{2xy}.

The force T is carried by the shear stress τ_y in the adhesive and the shear stresses τ_{1xy} and τ_{2xy} in the adherends. Their distributions for the adhesive joints shown in figures 4 and 5 are presented in figures 12-16. Large stress concentrations of τ_y occurring in the adherends of constant thickness are significantly reduced (6.1 to 7.5 times) if joints with sharp edges are used (figs. 17 and 18).

The reduction of stress concentrations τ_y and anchoring zone length in the joints loaded by shear forces are better viewed in the magnified stress profiles shown in figure 19. Contrary to the cases with axial forces or moments, anchoring zone lenghts are different now: $l_{\mathrm{anch}} \approx 0.6$ cm for the adherends with a constant thickness (figs. 19a, d) and $l_{\mathrm{anch}} \approx 1.6$ cm for the cases with sharp edges (figs. 19b, c, e, f).

There are no simple methods to assess the length of anchoring zones for the cases with shear forces.

The shear stress τ_{2xy} in the cross-sections $x = \pm l_x$ of the loaded adherend 2 have a parabolic distribution, corresponding to the loading in the form of shear forces $\mathsf{T} = \pm 1$ *N*. The maximal shear stress in adherend 2 is located at the X axis and equals $\tau_{2xy}(\pm\, l_x\,, 0) = -0.23438$ N/cm^2 (fig. 18a).

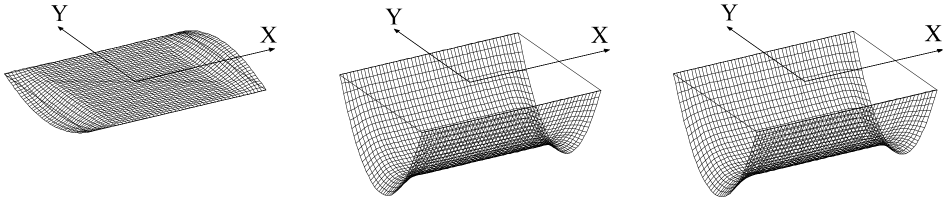


a) Stress τ_y in adhesive
$\tau_y(\pm l_x, 0) = \pm 0.22569$ N/cm^2

b) Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15612$ N/cm^2

c) Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15612$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 12. Stresses due to $\mathsf{T} = -1$ N in adhesive joint with covering plate of constant thickness as in figure 4a

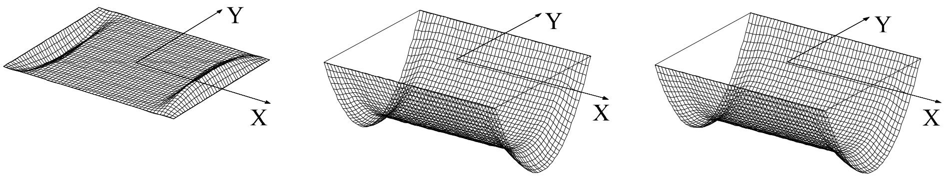


a) Stress τ_y in adhesive
$\tau_y(\pm l_x, 0) = \pm 0.03707$ N/cm^2

b) Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15608$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.18527$ N/cm^2

c) Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15608$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 13. Stresses due to T = –1 N in adhesive joint with covering plate of varying thickness and obtuse sharp edges as in figure 4b

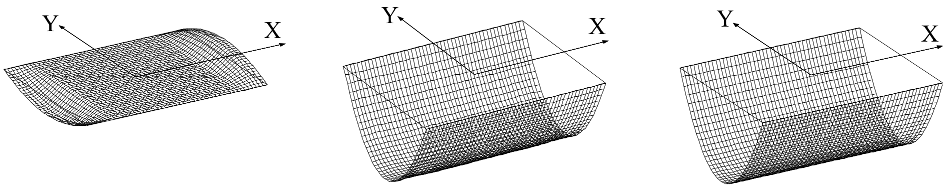


a) Stress τ_y in adhesive
max $|\tau_y| = 0.03569$ N/cm^2

b) Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15651$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.19247$ N/cm^2

c) Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15651$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 14. Stresses due to T = –1 N in adhesive joint with covering plate of varying thickness and tangential sharp edges as in figure 4c

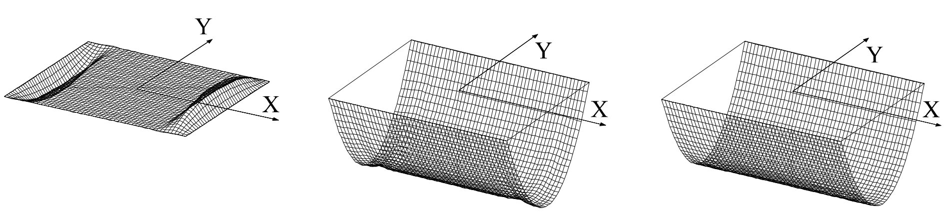


a) Stress τ_y in adhesive
$\tau_y(\pm l_x, 0) = \pm 0.036564$ N/cm^2

b) Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.1873$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.1864$ N/cm^2

c) Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.1873$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 15. Stresses due to T = –1 N in adhesive joint with insert of varying thickness and obtuse sharp edges as in figure 5b



a) Stress τ_y in adhesive
max $|\tau_y| = 0.043044$ N/cm^2

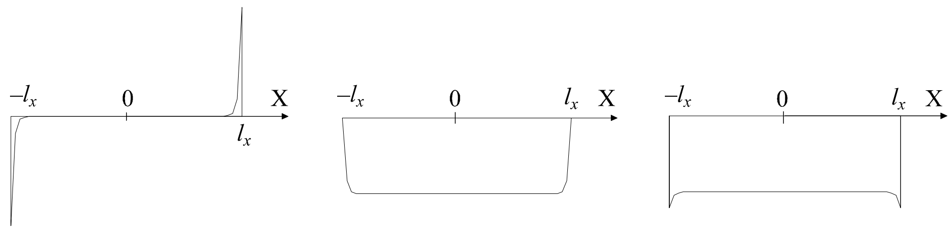
b) Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.18735$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.19116$ N/cm^2

c) Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.18735$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 16. Stresses due to T = –1 N in adhesive joint with insert of varying thickness and tangential sharp edges as in figure 5c

At the extreme cross-sections of adherend 1, which is not loaded, for the case of non-sharp edges the shear stress τ_{1xy} is zero, while for the sharp edges case, as unilateral internal boundaries, the stress varies from –0.18527 N/cm^2 to –0.19247 N/cm^2. These values are close to –0,1875 N/cm^2 which is the stress at the extreme cross-sections of adherend 2. In the case of covering plates or inserts with sharp edges, no shear stress concentrations τ_{1xy} are observed, contrary to the case of the normal stress concentrations σ_{1x} in joints loaded by axial forces or moments.

a) Adhesive joint with covering plate of constant thickness as in figure 4a

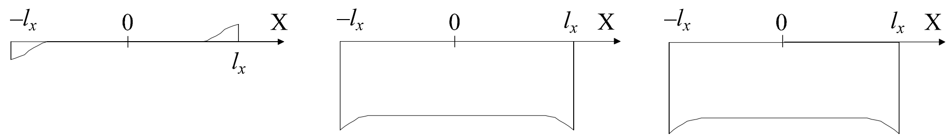


Stress τ_y in adhesive
$\tau_y(\pm l_x, 0) = \pm 0.22569$ N/cm^2

Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15612$ N/cm^2

Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15612$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

b) Adhesive joint with covering plate of varying thickness and obtuse sharp edges as in figure 4b

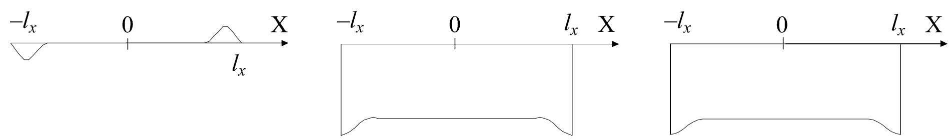


Stress τ_y in adhesive
$\tau_y(\pm l_x, 0) = \pm 0.03707$ N/cm^2

Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15608$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.18527$ N/cm^2

Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15608$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

c) Adhesive joint with covering plate of varying thickness and tangential sharp edges as in figure 4c



Stress τ_y in adhesive
max $|\tau_y| = 0.03569$ N/cm^2

Stress τ_{1xy} in adherend 1
$\tau_{1xy}(0, 0) = -0.15651$ N/cm^2
$\tau_{1xy}(\pm l_x, 0) = -0.19247$ N/cm^2

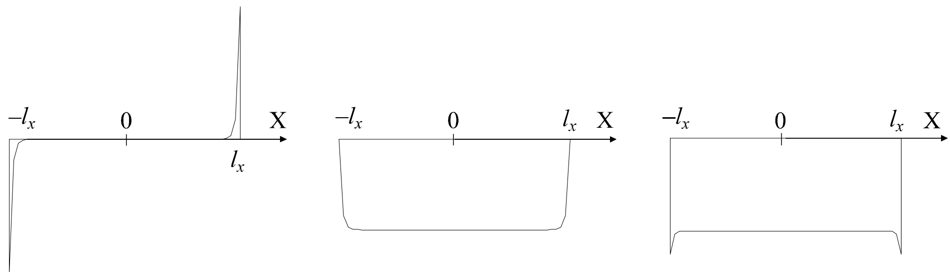
Stress τ_{2xy} in adherend 2
$\tau_{2xy}(0, 0) = -0.15651$ N/cm^2
$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 17. Stress profiles for shear force T = –1 N loading in adhesive joints with covering plates

The distributions of shear stresses τ_{1xy} and τ_{2xy} in the adherends in section $-l_x \leq x \leq l_x$ can be considered as parabolic, too. Stress profiles presented in figures 17 and 18 indicate the effect of displacement equilibration and gradual inclusion of adherends segments to the interaction along the anchoring zones for the cases with sharp edges.

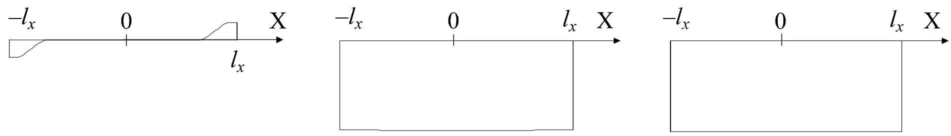
Shear stresses in direction X in adherends between anchoring zones, remain constant with a great level of accuracy. They have values differing by a fraction of a percent from those in a uniform element of the thickness g_2 = 1.2 cm in the reinforcement case or g_2 = 1.0 cm in the reconstruction case.

a) Adhesive joint with insert of constant thickness as in figure 5a



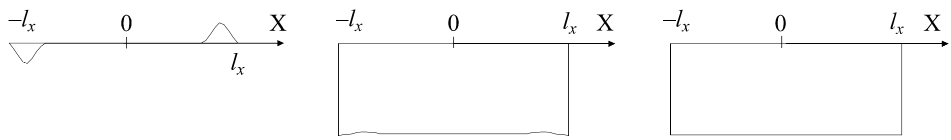
Stress τ_y in adhesive	Stress τ_{1xy} in adherend 1	Stress τ_{2xy} in adherend 2
$\tau_y(\pm l_x, 0) = \pm 0.27348$ N/cm^2	$\tau_{1xy}(0, 0) = -0.18734$ N/cm^2	$\tau_{2xy}(0, 0) = -0.18734$ N/cm^2
		$\tau_{2xy}(\pm l_x, 0) = -0.23438$ N/cm^2

b) Adhesive joint with insert of varying thickness and obtuse sharp edges as in figure 5b



Stress τ_y in adhesive	Stress τ_{1xy} in adherend 1	Stress τ_{2xy} in adherend 2
$\tau_y(\pm l_x, 0) = \pm 0.036564$ N/cm^2	$\tau_{1xy}(0, 0) = -0.1873$ N/cm^2	$\tau_{2xy}(0, 0) = -0.1873$ N/cm^2
	$\tau_{1xy}(\pm l_x, 0) = -0.1864$ N/cm^2	$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

c) Adhesive joint with insert of varying thickness and tangential sharp edges as in fig. 5c



Stress τ_y in adhesive	Stress τ_{1xy} in adherend 1	Stress τ_{2xy} in adherend 2
max $|\tau_y| = 0.043044$ N/cm^2	$\tau_{1xy}(0, 0) = -0.18735$ N/cm^2	$\tau_{2xy}(0, 0) = -0.18735$ N/cm^2
	$\tau_{1xy}(\pm l_x, 0) = -0.19116$ N/cm^2	$\tau_{2xy}(\pm l_x, 0) = -0.1875$ N/cm^2

Fig. 18. Stress profiles for shear force T = –1 N loading in adhesive joints with inserts

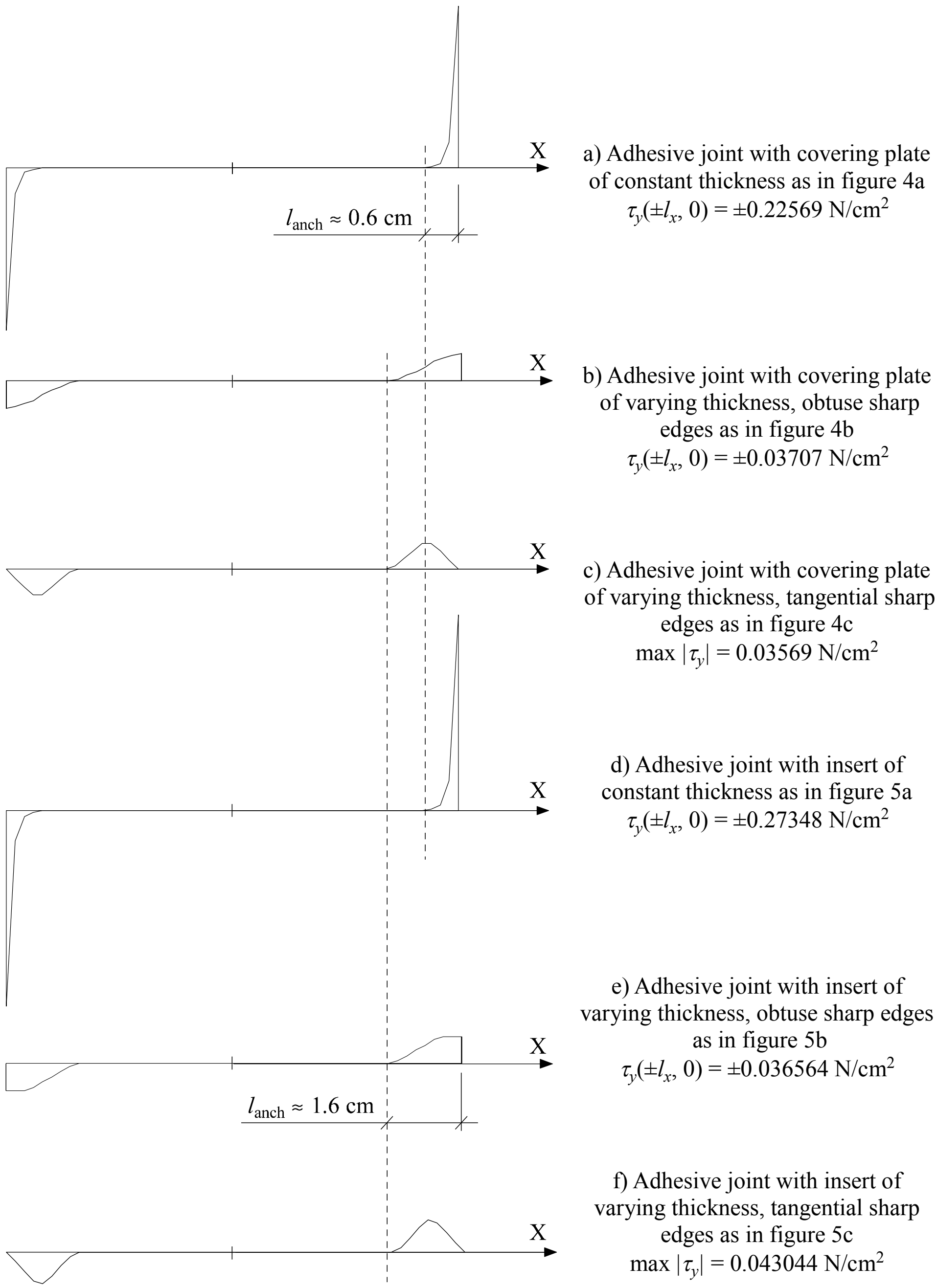


Fig. 19. Stress profiles τ_y in adhesive joints with covering plates and inserts loaded by shear forces $\mathbf{T} = \pm 1$ N. Graphical comparison of anchoring lengths

Conclusions

In adhesive joints loaded by moments, extreme values of the stress n_x in adhesive are found at edges of covering plates and inserts of constant thickness. The stress values n_x at corners of sharp edges are reduced by about 50-60%. The stress n_x in adhesive at tangential sharp edges is zero. Covering plates or inserts with tangential sharp edges take over the stresses from adherend 2 gradually, the maximum stress n_x in adhesive is found in the anchoring zone. Due to this fact, a risk of delamination at the edges is decreased. The maximal values of the stress n_x are smaller in this case than those at corners of obtuse sharp edges.

Adhesive joints with adherends of constant thickness feature the stress σ_{1x} in covering plates and inserts, which increases from zero at the edges $x = \pm l_x$ and rapidly reaches an approximately constant level between anchoring zones. In the loaded adherend 2 the stress σ_{2x} at the edges $x = \pm l_x$ assumes the boundary values, then decreases to a constant value, as in the adherend 1.

The level of stress σ_{1x} in adhesive joints with covering plates and inserts with obtuse sharp edges is constant and insignificantly exceeds the stress value of the load, there are only some small fluctuations at the anchoring zones.

The distribution of stress n_x in adhesive for adhesive joints with covering plates or inserts with tangential sharp edges is advantageous. However, a local increase of values of the stress σ_{1x} at the edges of covering plates and inserts is observed. This increase reaches about 100% of the mean stress value for a covering plate and about 60% for an insert. The reason for this local stress increase, is a small thickness of the adherend 1 at a sharp edge.

In all variants of adhesive joints loaded by moments, stress in adhesive between anchoring zones is close to or equal to zero.

On the other hand normal stresses in adherends between anchoring zones, feature distributions approximately linear and coinciding with ±0,3% accuracy with the normal stress distribution in an element subjected to bending.

Shear stress concentrations τ_y in adhesive for adhesive joints loaded by a constant shear force, are located in anchoring zones and are of similar character as those for stress τ_x in the cases of loading by moments. The stresses τ_y in adhesive for all the cases of joints loaded by a shear force are close to or equal to zero between the anchoring zones. In the anchoring zones shear stress concentrations τ_{xy} in adherends are smaller than normal stress concentrations for joints under bending and are practically non-existent for joints with sharp edges. Shear stress distributions τ_{xy} for the adherends coincide with a 0.2% accuracy with the shear stress distribution for a uniform element subjected to shearing.

The comparison of stress distributions allows us to conclude, that the use of covering plates or inserts with obtuse sharp edges can be an effective method of reinforcement or reconstruction of a weakened element.

The problems presented in this paper were solved using the author's computer program SPOINA (ADHESIVE). The program, based on the formulae derived by the author, solves the set of four elliptic partial differential equations of the second order by means of a finite-difference method and presents the results in table and graph form.

References

Ahn S.H., Springer G.S. [2000]: Repair of composite laminates. U.S. Department of Transportation Federal Aviation Administration, Office of Aviation Research Washington, D.C. 20591, Final Report DOT/FAA/AR-00/46

Bahei-El-Din Y.A., Dvorak G.J. [2001]: New design of adhesive joints for thick composite laminates. Composites Science and Technology 61 [1]: 19-40

Boss J.N., Ganesh V.K., Lim C.T. [2003]: Modulus grading versus geometrical grading of composite adherends in single-lap bonded joints. Composite Structures 62 [1]: 113-121

Cea J. [1964]: Approximation variationelle des problemes aux limites (Variational approximation of boundary problems). Annales de l'institut Fourier 14 [2]: 345-388

Fichera G. [1972]: Existence theorems in elasticity. In: Flügge S. (ed.). Encyclopedia of Physics Vol. VIa/2. Mechanics of Solids II. Springer-Verlag, Berlin

Forsythe G.E., Wasow W.R. [1960]: Finite-difference methods for partial differential equations. John Wiley & Sons, Inc, New York-London

Kaye R.H., Heller M. [2002]: Through-thickness shape optimization of bonded repairs and lap-joints. International Journal of Adhesion & Adhesives 22 [1]: 7-21

Kumar S.B., Sridhar I., Sivashanker S., Osiyemi S.O., Bag A. [2006]: Tensile failure of adhesively bonded CFRP composite scarf joints. Materials Science and Engineering B 132 [12]: 113-120

Neuhaus H. [1994]: Lehrbuch des Ingenieurholzbaus. B.G. Teubner, Stuttgart-Leipzig

Rapp P. [2015a]: Mechanics of adhesive joints as a plane problem of the theory of elasticity. Part II: Displacement formulation for orthotropic adherends. Archives of Civil and Mechanical Engineering 15 [2]: 603-619

Rapp P. [2015b]: Methodology and examples of revalorization of wooden structures in historic buildings. Wiadomości Konserwatorskie – Journal of Heritage Conservation 43: 92-108

Rapp P. [2016]: Application of adhesive joints in reinforcement and reconstruction of weakened wooden elements loaded axially. Drewno 59 [196]: 59-73. DOI: 10.12841/wood.1644-3985.128.05

Submission date: 04.11.2016

Online publication date: 13.07.2017